

# 1. QUARK MODEL

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## 1.1. Quantum numbers of the quarks

Quarks are strongly interacting fermions with spin 1/2 and, by convention, positive parity. Antiquarks have negative parity. Quarks have the additive baryon number 1/3, antiquarks -1/3. Table 14.1 gives the other additive quantum numbers (flavors) for the three generations of quarks. They are related to the charge  $Q$  (in units of the elementary charge  $e$ ) through the generalized Gell-Mann-Nishijima formula

$$Q = I_z + \frac{B + S + C + B + T}{2}, \quad (1.1)$$

where  $B$  is the baryon number. The convention is that the *flavor* of a quark ( $I_z$ ,  $S$ ,  $C$ ,  $B$ , or  $T$ ) has the same sign as its *charge*  $Q$ . With this convention, any flavor carried by a charged meson has the same sign as its charge, *e.g.*, the strangeness of the  $K^+$  is +1, the bottomness of the  $B^+$  is +1, and the charm and strangeness of the  $D_s^-$  are each -1. Antiquarks have the opposite flavor signs.

## 1.2. Mesons

Mesons have baryon number  $B = 0$ . In the quark model, they are  $q\bar{q}'$  bound states of quarks  $q$  and antiquarks  $\bar{q}'$  (the flavors of  $q$  and  $q'$  may be different). If the orbital angular momentum of the  $q\bar{q}'$  state is  $\ell$ , then the parity  $P$  is  $(-1)^{\ell+1}$ . The meson spin  $J$  is given by the usual relation  $|\ell - s| \leq J \leq |\ell + s|$ , where  $s$  is 0 (antiparallel quark spins) or 1 (parallel quark spins). The charge conjugation, or  $C$ -parity  $C = (-1)^{\ell+s}$ , is defined only for the  $q\bar{q}$  states made of quarks and their own antiquarks. The  $C$ -parity can be generalized to the  $G$ -parity  $G = (-1)^{I+\ell+s}$  for mesons made of quarks and their own antiquarks (isospin  $I_z = 0$ ), and for the charged  $u\bar{d}$  and  $d\bar{u}$  states (isospin  $I = 1$ ).

The mesons are classified in  $J^{PC}$  multiplets. The  $\ell = 0$  states are the pseudoscalars ( $0^{-+}$ ) and the vectors ( $1^{--}$ ). The orbital excitations  $\ell = 1$  are the scalars ( $0^{++}$ ), the axial vectors ( $1^{++}$ ) and ( $1^{+-}$ ), and the tensors ( $2^{++}$ ). Assignments for many of the known mesons are given in Tables 14.2 and 14.3. Radial excitations are denoted by the principal quantum number  $n$ . The very short lifetime of the  $t$  quark makes it likely that bound-state hadrons containing  $t$  quarks and/or antiquarks do not exist.

States in the natural spin-parity series  $P = (-1)^J$  must, according to the above, have  $s = 1$  and hence,  $CP = +1$ . Thus, mesons with natural spin-parity and  $CP = -1$  ( $0^{+-}$ ,  $1^{-+}$ ,  $2^{+-}$ ,  $3^{-+}$ , *etc.*) are forbidden in the  $q\bar{q}'$  model. The  $J^{PC} = 0^{--}$  state is forbidden as well. Mesons with such *exotic* quantum numbers may exist, but would lie outside the  $q\bar{q}'$  model (see section below on exotic mesons).

Following  $SU(3)$ , the nine possible  $q\bar{q}'$  combinations containing the light  $u$ ,  $d$ , and  $s$  quarks are grouped into an octet and a singlet of light quark mesons:

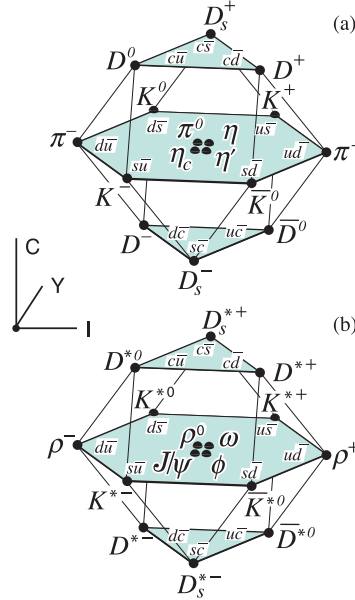
$$3 \otimes \bar{3} = 8 \oplus 1. \quad (1.2)$$

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A fourth quark such as charm  $c$  can be included by extending SU(3) to SU(4). However, SU(4) is badly broken owing to the much heavier  $c$  quark. Nevertheless, in an SU(4) classification, the sixteen mesons are grouped into a 15-plet and a singlet:

$$4 \otimes \bar{4} = 15 \oplus 1. \quad (1.3)$$

The *weight diagrams* for the ground-state pseudoscalar ( $0^{-+}$ ) and vector ( $1^{--}$ ) mesons are depicted in Fig. 14.1. The light quark mesons are members of nonets building the middle plane in Fig. 14.1(a) and (b).



**Figure 1.1:** SU(4) weight diagram showing the 16-plets for the pseudoscalar (a) and vector mesons (b) made of the  $u$ ,  $d$ ,  $s$ , and  $c$  quarks as a function of isospin  $I$ , charm  $C$ , and hypercharge  $Y = S + B - \frac{C}{3}$ . The nonets of light mesons occupy the central planes to which the  $c\bar{c}$  states have been added.

Isoscalar states with the same  $J^{PC}$  will mix, but mixing between the two light quark isoscalar mesons, and the much heavier charmonium or bottomonium states, are generally assumed to be negligible.

### 1.3. Baryons: $qqq$ states

Baryons are fermions with baryon number  $B = 1$ , *i.e.*, in the most general case, they are composed of three quarks plus any number of quark - antiquark pairs. Although recently some experimental evidence for  $(qqqq\bar{q})$  pentaquark states has been claimed (see review on Possible Exotic Baryon Resonance), so far all established baryons are 3-quark ( $qqq$ ) configurations. The color part of their state functions is an SU(3) singlet, a completely antisymmetric state of the three colors. Since the quarks are fermions, the state function must be antisymmetric under interchange of any two equal-mass quarks (up and down quarks in the limit of isospin symmetry). Thus it can be written as

$$|qqq\rangle_A = |\text{color}\rangle_A \times |\text{space, spin, flavor}\rangle_S, \quad (1.21)$$

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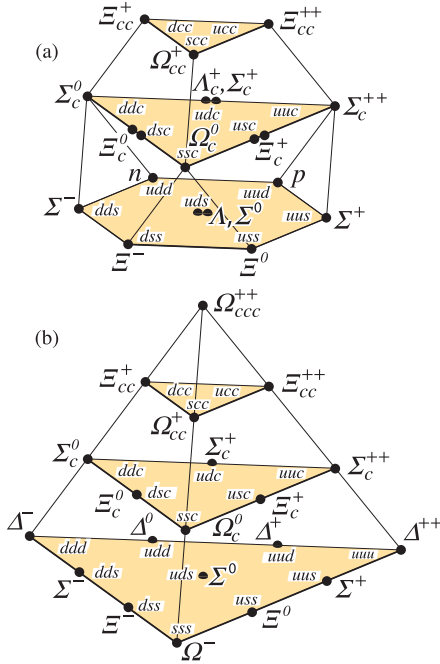
where the subscripts  $S$  and  $A$  indicate symmetry or antisymmetry under interchange of any two equal-mass quarks. Note the contrast with the state function for the three nucleons in  ${}^3\text{H}$  or  ${}^3\text{He}$ :

$$|NNN\rangle_A = |\text{space, spin, isospin}\rangle_A . \quad (1.22)$$

This difference has major implications for internal structure, magnetic moments, *etc.*

The “ordinary” baryons are made up of  $u$ ,  $d$ , and  $s$  quarks. The three flavors imply an approximate flavor  $\text{SU}(3)$ , which requires that baryons made of these quarks belong to the multiplets on the right side of

$$\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} = \mathbf{10}_S \oplus \mathbf{8}_M \oplus \mathbf{8}_A \oplus \mathbf{1}_A . \quad (1.23)$$



**Figure 1.2:**  $\text{SU}(4)$  multiplets of baryons made of  $u$ ,  $d$ ,  $s$ , and  $c$  quarks. (a) The 20-plet with an  $\text{SU}(3)$  octet. (b) The 20-plet with an  $\text{SU}(3)$  decuplet.

Here the subscripts indicate symmetric, mixed-symmetry, or antisymmetric states under interchange of any two quarks. The  $\mathbf{1}$  is a  $uds$  state ( $\Lambda_1$ ), and the octet contains a similar state ( $\Lambda_8$ ). If these have the same spin and parity, they can mix. The mechanism is the same as for the mesons (see above). In the ground state multiplet, the  $\text{SU}(3)$  flavor singlet  $\Lambda_1$  is forbidden by Fermi statistics. Section 41, on “ $\text{SU}(3)$  Isoscalar Factors and Representation Matrices,” shows how relative decay rates in, say,  $\mathbf{10} \rightarrow \mathbf{8} \otimes \mathbf{8}$  decays may be calculated. The addition of the  $c$  quark to the light quarks extends the flavor symmetry to  $\text{SU}(4)$ . However, due to the large mass of the  $c$  quark, this symmetry is much more strongly broken than the  $\text{SU}(3)$  of the three light quarks. Figures 1.2(a) and 1.2(b) show the  $\text{SU}(4)$  baryon multiplets that have as their bottom levels an  $\text{SU}(3)$  octet, such as the octet that includes the nucleon, or an  $\text{SU}(3)$  decuplet, such as the decuplet that includes the  $\Delta(1232)$ . All particles in a given

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SU(4) multiplet have the same spin and parity. The charmed baryons are discussed in more detail in the “Note on Charmed Baryons” in the Particle Listings. The addition of a  $b$  quark extends the flavor symmetry to SU(5); the existence of baryons with  $t$ -quarks is very unlikely due to the short lifetime of the top.

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For further details, see the full *Review of Particle Physics*.